【Grant-in-Aid for Transformative Research Areas（Ｂ）】

A Study of Mathematical Shape Design
(Shape Design Modeling based on the Mathematics)

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Research Area Information	Number of Research Area : 23B201 Project Period (FY) : 2023-2025 Keywords : Shape Design, Topology optimization, PDE, Mathematical Model	

Purpose and Background of the Research

●Outline of the Research

Based on collaboration between researchers in the fields of mechanical engineering and mathematics, this research project proposes a new academic discipline for shape design, "Shape Design Mathematics," which makes full use of state-of-the-art mathematics. If such an academic discipline can be constructed, quantitative evaluation of the relationship between the geometrical features of a shape and each mechanical property will become possible, enabling truly innovative shape design of mechanical products and devices while grounded in mechanics and mathematics. This will bring about a major revolution in manufacturing-related fields centered on mechanical engineering.

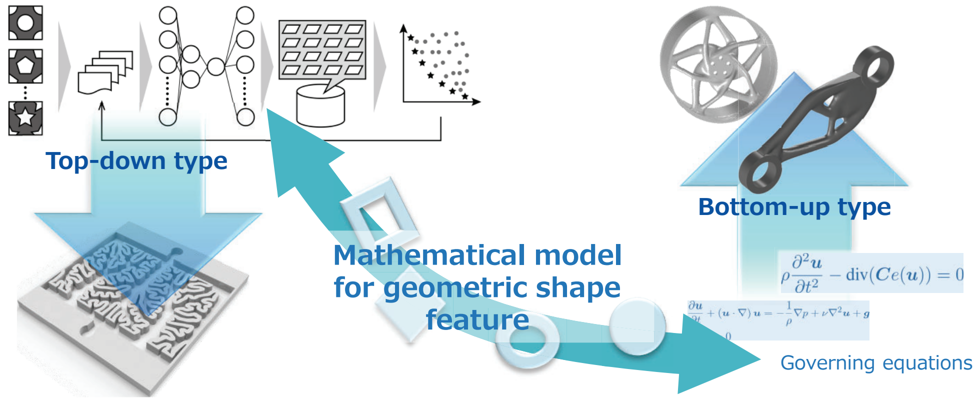

Figure 1. overview

●Background

The field of mechanical engineering is the study of manufacturing based on fluid mechanics, thermodynamics, material mechanics, and mechanical dynamics. Shape design modeling based on mechanics is essential for designing mechanical products with new added value and functionality, but there are limitations in design that focuses only on specific mechanical properties. On the other hand, with the recent dramatic improvement in computer performance, digital design technology using numerical analysis and machine learning has attracted much attention. However, although each mechanical discipline has common mathematical structures, they have not yet been systematically organized to build a unified and systematic mathematical foundation for shape design problems in mechanical engineering. As a result, there has been no systematization of shape design modeling methods that organically link the knowledge of each mechanical discipline and mathematics, and the establishment of a design theory to create truly innovative mechanical products has not yet been achieved.

●Objective

This project aims to systematize mathematics-based shape design modeling for shape design problems in mechanical engineering. Specifically, we broadly classify design problems into those that can be approximated linearly and those that are dominated by nonlinearity. For the former, we will develop a bottom-up shape design modeling method based on analytical methods such as asymptotic analysis, and for the latter, a data-driven top-down shape design modeling method based on information geometry. We will also develop a middle-up-down method that identifies design shape parameters with truly useful geometric features for each design problem and enables organic coordination of both modeling methods based on geometric features. These methods will enable quantitative evaluation of the relationship between models and geometric features for each mechanical property.

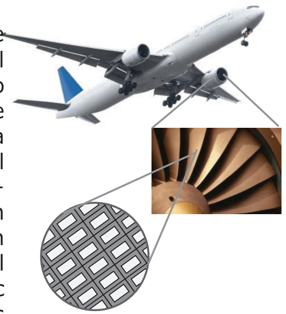

Figure 2. Shape design problems at various scales

Expected Research Achievements

●A01: Bottom-up type

We develop shape design modeling methods that make full use of mathematical and analytical approches. Starting from the methodology of evolving sensitivity analysis methods such as asymptotic expansion and topological derivatives, we will extend the method to vector elliptic equations and time evolution problems. Furthermore, homogenization methods for curved geometries and asymptotic analysis of curved geometries with the perspective of random matrix theory of the discrete Laplacian for application to machine learning will be conducted in collaboration with Group A02. In collaboration with Group A03, we will also develop a mathematical basis for shape design problems coupled with partial differential equations for geometric features.

●A02: Top-down type

We develop a data-driven shape design modeling method that makes full use of information geometry. Furthermore, by organically integrating a generative model based on machine learning and a methodology for optimal solution search with high design degrees of freedom, we will enable exhaustive solution search in the design solution space and indirectly derive the desired design solution even for complex shape design problems. In collaboration with Group A03, we will also formulate the problem as a multi-physics shape design problem with shape design problems based on geometric features, enabling organic linkage with shape design problems based on mathematical analysis methods by Group A01. The effectiveness and validity of the shape design modeling method will be verified through case studies of fluid structure design problems with strong nonlinearity, and the shape design modeling integrated with Group A03 and Group A01 will be constructed and compared.

●A03: middle-up-down type

We will construct mathematical models that enable comprehensive evaluation of geometric features of shape design solutions, and develop a comprehensive shape design modeling method that seamlessly connects each shape design problem and its production. Specifically, we will construct a mathematical model system that can be evaluated centrally based on geometry and analysis, and develop a method for setting essentially useful design variables for each model constructed by Groups A01 and A02. Furthermore, in order to make design modeling that takes into account the production process, a method for mathematically integrating each shape design model based on geometric conditions will be formulated using partial differential equations for space. In the verification through case studies, the team will be in charge of verifying the effectiveness of the method as a modeling method from design to production, mainly in cooperation with the A02 group.

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