Section 3 Mathematics

I. OVERALL OBJECTIVES

Through mathematical activities, to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, to help students acquire the way of mathematical representation and processing, to develop their ability to think and represent phenomena mathematically, to help students enjoy their mathematical activities and appreciate the value of mathematics, and to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging.

II. OBJECTIVES AND CONTENT FOR EACH GRADE

[Grade 1]

1. Objectives

   (1) To expand the range of numbers to positive and negative numbers, and to deepen students' understanding of the concept of numbers. To understand the necessity and meaning of the use of letters and of equations, to cultivate students' ability to represent and process the relationships and rules of numbers and quantities in a general and simple manner, and to make use of linear equations with one unknown.

   (2) Through activities like observation, manipulation and experimentation with plane and space figures, to deepen students' intuitive ways of viewing and thinking about geometrical figures and to cultivate their ability to think and represent logically.

   (3) Through examining concrete phenomena, to deepen students' understanding of direct proportion and inverse proportion, and to cultivate their ability to find out, represent and think functional relationships.

   (4) To cultivate students' ability to collect and organize data according to their purpose, then read trends in the data.

2. Content

   A. Numbers and Algebraic Expressions

   (1) To be able to understand positive and negative numbers in concrete situations and to perform the four fundamental operations with these numbers, and to be able to use represent and consider by using positive and negative numbers.

   (a) To understand the necessity and meaning of positive and negative numbers.

      (b) To understand the meaning of four fundamental operations with positive and negative numbers through relating the meaning to the four fundamental operations with numbers learned in elementary school.

      (c) To perform the four fundamental operations with positive and negative numbers.

      (d) To represent and process by using positive and negative numbers in concrete situations.

(2) To cultivate the ability to represent the relationships and rules of numbers and quantities in algebraic expressions using letters, to read the meaning of these expressions, and to be able to calculate algebraic expressions using letters.

      (a) To understand the necessity and meaning of using letters.

      (b) To know how to express multiplication and division in algebraic expressions using letters.

      (c) To calculate addition and subtraction with simple linear expressions.

      (d) To understand how to represent the relationships and rules of numbers and quantities in algebraic expressions using letters, and to represent and read by using the expressions.

(3) To understand equations, and to be able to consider by using linear equations with one unknown.

      (a) To understand the necessity and meaning of equations, as well as the meanings of letters within equations and their solutions.

      (b) To know how to solve equations based on the properties of equalities.

      (c) To solve simple linear equations with one unknown and make use of the linear equations in concrete situations.

[Terms and Symbols]

    natural number, sign, absolute value, term, coefficient, transposing terms,   ,

  B. Geometrical Figures

(1) Through activities like observation, manipulation and experimentation, to deepen the understanding of plane figures by constructing them with a prospect in mind and exploring the relationship between geometrical figures, and to cultivate the ability to think and represent logically.

      (a) To understand the fundamental methods for constructing figures like bisector of an angle, perpendicular bisector of a line segment and perpendicular lines, and to make use of them in concrete situations.

      (b) To understand parallel translation, symmetric transformation and

rotational transformation, and to explore the relationship between two geometrical figures.

(2) Through activities like observation, manipulation and experimentation, to deepen the understanding of space figures, and to develop the ability to measure geometrical figures.

(a) To know the positional relationship between straight lines and planes in space.

(b) To grasp space figures as objects constructed by the movement of straight lines and plane figures, and to represent space figures on a plane and read their properties in the representation.

(c) To find the length and area of a sector, as well as the surface area and volume of basic cylinder, pyramid and sphere.

[Terms and Symbols]

arc, chord, solid of revolution, skewed position, π, //, $\perp$, $\angle$, $\triangle$

C. Functions

(1) Through finding out two numbers/quantities in concrete phenomena and exploring their change and correspondence, to deepen the understanding of relationships of direct proportion and inverse proportion, and to cultivate the ability to find out, represent and think functional relationships.

(a) To understand the meaning of functional relationships.

(b) To understand the meaning of direct proportion and inverse proportion.

(c) To understand the meaning of coordinates.

(d) To represent direct proportion and inverse proportion into tables, algebraic expressions, graphs and so on, and to understand their characteristics.

(e) To grasp and explain concrete phenomena by using direct proportion and inverse proportion.

[Terms and Symbols]

function, variable, domain

D. Making Use of Data

(1) To be able to collect data according to a purpose, arrange it into tables and graphs by using a computer and other means, and then read trends in the data focusing on its representative values and its variations.

(a) To understand the necessity and meaning of histogram and representative values.

(b) To grasp and explain trends in the data by using histograms and

representative values.

[Terms and Symbols]

mean, median, mode, relative frequency, range, class

[Mathematical Activities]

(1) In learning each content of "A. Numbers and Algebraic Expressions," "B. Geometrical Figures," "C. Functions," and "D. Making Use of Data,", and in learning the connections of these contents, students should be provided their opportunities doing mathematical activities like the following.

(a) Activities for finding out the properties of numbers and geometrical figures based on previously learned mathematics

(b) Activities for making use of mathematics in daily life

(c) Activities for explaining and communicating each other in one's own way by using mathematical representations

3. Handling the Content

(1) In connection with (1) under "A. Numbers and Algebraic Expressions" in "Content," the concept of number sets and the possibility of the four fundamental operations should be dealt with.

(2) In connection with (2)-(d) under "A. Numbers and Algebraic Expressions" in "Content," representing magnitude relationships by using inequalities should be dealt with.

(3) In connection with (3)-(c) under "A. Numbers and Algebraic Expressions" in "Content," solving simple proportional equations should be dealt with.

(4) In connection with (1)-(a) under "B. Geometrical Figures" in "Content," the fact that any tangent of a circle is perpendicular to the radius passing through its point of tangency should be dealt with.

(5) In connection with (2)-(b) under "B. Geometrical Figures" in "Content," sketch, developed drawing and projection drawing should be dealt with.

(6) In connection with (1) under "D. Making Use of Data" in "Content," calculation errors, approximate values and $a \times 10^n$ format expression should be dealt with.

[Grade 2]

1. Objectives

(1) To foster students' ability to calculate and transform algebraic expressions using letters according to their purpose, and to cultivate their ability to understand and use simultaneous linear equations with two unknowns.

(2) Through activities like observation, manipulation and experimentation, to deepen

students' understanding of the properties of basic plane figures, and to understand the necessity, meaning and methods of mathematical reasoning in considerations of the properties of geometrical figures, and to foster their ability to think and represent logically.

(3) Through exploring concrete phenomena, to understand linear functions, and to foster students' ability to find out, represent and think about functional relationships.

(4) To cultivate students' ability to understand and use probability through exploring uncertain phenomena.

2. Content

A. Numbers and Algebraic Expressions

(1) To foster the ability to find out relationships of numbers and quantities in concrete phenomena, represent these relationships in algebraic expressions using letters and read the meaning of these expressions, and to be able to calculate the four fundamental operations with expressions using letters.

(a) To calculate addition and subtraction with simple polynomials, as well as multiplication and division with monomials.

(b) To understand how to grasp and explain numbers and quantities and the relationships of numbers and quantities in algebraic expressions using letters.

(c) To transform simple algebraic expressions according to the purpose.

(2) To understand simultaneous linear equations with two unknowns, and to be able to consider by using it.

(a) To understand the meaning of linear equations with two unknowns and their solutions.

(b) To understand the necessity and meaning of simultaneous linear equations with two unknowns and the meaning of their solutions.

(c) To solve simple simultaneous linear equations with two unknowns and make use of them in concrete situations.

[Terms and Symbols]

similar term

B. Geometrical Figures

(1) Through activities like observation, manipulation and experimentation, to be able to find out the properties of basic plane figures and verify them based on the properties of parallel lines.

(a) To understand the properties of parallel lines and angles and basing on it,

to verify and explain the properties of geometrical figures.

(b) To know how to find out the properties of angles of polygons based on the properties of parallel lines and angles of triangle.

(2) To understand the congruence of geometrical figures and deepen the way of viewing geometrical figures, to verify the properties of geometrical figures based on the facts like the conditions for congruence of triangles, and to foster the ability to think and represent logically.

(a) To understand the meaning of congruence of plane figures and the conditions for congruence of triangles.

(b) To understand the necessity, meaning and methods of proof.

(c) To logically verify the basic properties of triangles and parallelograms based on the facts like the conditions for congruence of triangles, and to find out new properties by reading proofs of the properties of geometrical figures.

[Terms and Symbols]

opposite angle, interior angle, exterior angle, definition, proof, converse, $\equiv$

C. Functions

(1) Through finding out two numbers/quantities in concrete phenomena and exploring their change and correspondence, to understand linear functions, and to foster the ability to find out, represent and think about functional relationships.

(a) To know that in concrete phenomena there are some phenomena which can be grasped as linear functions.

(b) To understand linear functions by interrelating their tables, algebraic expressions and graphs.

(c) To recognize linear equations with two unknowns as algebraic expressions representing functions.

(d) To grasp and explain concrete phenomena by using linear functions.

[Terms and Symbols]

rate of change, slope

D. Making Use of Data

(1) Through activities like the observation of and experimentation on uncertain phenomena, to understand probability, and to be able consider and represent by using probability.

(a) To understand the necessity and meaning of probability, and find the probability of an uncertain event in simple cases.

(b) To grasp and explain uncertain phenomena by using probability.

[Mathematical Activities]

  (1) In learning each content of "A. Numbers and Algebraic Expressions," "B. Geometrical Figures," "C. Functions," and "D. Making Use of Data,", and in learning the connection of these contents, students should be provided their opportunities for doing mathematical activities like the following.

    (a) Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics

    (b) Activities for making use of mathematics in daily life and society

    (c) Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations

3. Handling the Content

  (1) In connection with (2)-(c) under "B. Geometrical Figures" in "Content," the fact that square, rhombus and rectangle are special cases of parallelogram should be dealt with.

[Grade 3]

1. Objectives

  (1) To understand the square roots of positive numbers and deepen students' understanding of the concept of numbers. To develop students' ability to calculate and transform algebraic expressions according to the purpose, and to cultivate students' ability to understand and use quadratic equations.

  (2) Through activities like observation, manipulation and experimentation, to understand the similarity of geometrical figures, the relationships between inscribed angles and its central angle in a circle, and the Pythagorean Theorem, and to develop students' ability to use them in thinking about the properties of geometrical figures and measuring geometrical figures, as well as their ability to logically think about and represent geometrical figures with a prospect in mind.

  (3) Through exploring concrete phenomena, to understand the function $y = ax^2$, and to develop students' ability to find out, represent and think about functional relationships.

  (4) To cultivate students' ability to read trends in a population by selecting samples out of the population and exploring its trends.

2. Content

  A. Numbers and Algebraic Expressions

(1) To understand the square roots of positive numbers, and to be able to represent and consider by using the square roots.

    (a) To understand the necessity and meaning of square roots of positive numbers.

    (b) To calculate simple expressions which include square roots of positive numbers.

    (c) To represent and process by using square roots of positive numbers in concrete situations.

(2) To be able to expand and factor simple polynomial expressions using letters, and to develop students' ability to transform algebraic expressions and read their meanings according to the purpose.

    (a) To perform multiplication of monomial and polynomial expressions, and division of a polynomial expression by a monomial expression.

    (b) To perform multiplication of simple linear expressions and expand and factor simple algebraic expressions using the following formulas:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(a - b) = a^2 - b^2$$
$$(x + b)(x + b) = x^2 + (a + b)x + ab$$

    (c) To grasp and explain numbers and quantities and the relationships of numbers and quantities through algebraic expressions using letters.

(3) To understand quadratic equations, and to be able to consider by using quadratic equations.

    (a) To understand the necessity and meaning of quadratic equations and the meaning of their solutions.

    (b) To solve quadratic equations by factoring them and transforming them to the squared expressions.

    (c) To know the solution formula for quadratic equations and to solve quadratic equations by using the formula.

    (d) To make use of quadratic equations in concrete situations.

[Terms and Symbols]

    radical sign, rational number, irrational number, factor, $\sqrt{}$

B. Geometrical Figures

(1) To verify the properties of geometrical figures based on the facts like the conditions for similar triangles, to develop the ability to think and represent logically, and to be able to consider by using the properties of similar

geometrical figures.

(a) To understand the meaning of the similarity of plane figures and the conditions for similar triangles.

(b) To verify logically the basic properties of geometrical figures based on the facts like the conditions for similar triangles.

(c) To find out and verify the properties of ratio of line segments related to parallel lines.

(d) To understand the meaning of the similarity of basic solids, as well as the relationships between the scale factor, the ratio of areas and the ratio of volumes of similar geometric figures.

(e) To make use of the properties of similar geometrical figures in concrete situations.

(2) Through activities like observation, manipulation and experimentation, to find out and understand the relationships between inscribed angles and its central angle in a circle, and to be able to consider by using the relationships.

(a) To understand the meaning of the relationships between inscribed angles and its central angle in a circle, and to know that the relationships can be proven.

(b) To make use of the relationships between inscribed angles and its central angle in concrete situations.

(3) Through activities like observation, manipulation and experimentation, to find out and understand the Pythagorean Theorem, and to be able to consider by using the theorem.

(a) To understand the meaning of the Pythagorean Theorem, and to know that the theorem can be proven.

(b) To make use of the Pythagorean Theorem in concrete situations.

[Terms and Symbols]

C. Functions

(1) Through finding out two numbers/quantities in concrete phenomena and exploring their changes and correspondence, to understand the function $y = ax^2$, and to develop the ability to find out, represent and think about functional relationships.

(a) To know that there are some phenomena which can be grasped as the function $y = ax^2$ in concrete phenomena.

(b) To understand the function $y = ax^2$ by interrelating their tables,

algebraic expressions and graphs.

(c) To grasp and explain concrete phenomena by using the function $y = ax^2$.

(d) To understand that in various concrete phenomena there are some functional relationships.

D. Making Use of Data

(1) Through selecting samples out of a population and exploring its trends by using a computer and other means, to be able to understand that it is possible to read trends in the population.

(a) To understand the necessity and meaning of a sample survey.

(b) To carry out sample surveys in simple cases, and grasp and explain trends in the population.

[Terms and Symbols]

complete count survey

[Mathematical Activities]

(1) In learning each content of "A. Numbers and Algebraic Expressions," "B. Geometrical Figures," "C. Functions," and "D. Making Use of Data,", and in learning the connection of these contents, students should be provide their opportunities for doing mathematical activities like the following.

(a) Activities for finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics

(b) Activities for making use of mathematics in daily life and society

(c) Activities for explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations

3. Handling the Content

(1) In connection with items like (2) under "A. Numbers and Algebraic Expressions" in "Content," factoring natural numbers into the product of prime numbers should be dealt with.

(2) In connection with (3) under "A. Numbers and Algebraic Expressions" in "Content," quadratic equations with real number solutions should be dealt with.

(3) In connection with (3)-b under "A. Numbers and Algebraic Expressions" in "Content," the quadratic equations $ax^2 = b$ (where a and b are rational numbers) and the quadratic equations $x^2 + px + q = 0$ (where p and q are integers) should be dealt with. The teaching of how to solve quadratic equations by factoring them should deal mainly with the quadratic equations where the formulas shown in (2)-b under "A. Numbers and Algebraic Expressions" in "Content" can be used.

In addition, the teaching of how to solve the quadratic equations by transforming them to the squared expressions should deal mainly with the quadratic equations where the coefficient of x is an even number.

(4) In connection with (2) under "B. Geometrical Figures" in "Content," the converse of the theorem of inscribed angles should be dealt with.

III. SYLLABUS DESIGN AND HANDLING THE CONTENT

1. In designing the syllabus, consideration should be given to the following:

   (1) Some of the contents for each of the grades can be dealt with lightly, to the extent that this treatment does not hinder the achievement of the objectives for the grade in Section II, with the instructional content being provided in upper grades. In addition, the teaching can be carried out by adding some contents from upper grades, to the extent that this treatment does not deviate from the objectives for the grade.

   (2) In order to ensure reliable learning on the part of the students, when teaching a new content, consideration should be given to intentionally taking up related content that has already been taught once again and providing opportunities to relearn the content.

   (3) Based on the objectives of moral education listed in Sections I-2 of Chapter 1 "General Provisions" and in Section I of Chapter 3 "Moral Education", instructions concerning the content listed in Section II of Chapter 3 "Moral Education" should be given appropriately. The instructions should be in accordance with the characteristics of mathematics and should be related to the period for moral education.

2. In the handling the content listed in Section II, consideration should be given to the following:

   (1) Terms and symbols indicated under the content for each grade in Section II have the purpose of clarify the range and the extent of the content dealt with in each grade. In teaching of these terms and symbols, consideration should be given to dealing with them in close connection with the content for each grade.

   (2) In teaching the content of each area, consideration should be given to properly using tools like *soroban* (Japanese abacus), calculators, computers, and information and communication networks as needed in order to improve the learning results. This should especially be taken into account for the instructional content related to numerical calculations, as well as in teaching through activities like observation, manipulation and experimentation.

3. In teaching mathematical activities, consideration should be given to the following:
   (1) Mathematical activities should be enjoyable for students, and opportunities should be created in order to realize the significance of learning mathematics and the necessity of mathematics.
   (2) Opportunities should be given to students for finding out one's own problems to be solved, making a plan for solving them, implementing the plan, and then evaluating and improving the results.
   (3) Opportunities should be given to students for sharing among others the results of mathematical activities by looking back at the process of mathematical activities, and then making and presenting their reports on the activities.
4. The "problem situation-based learning" is a learning which solves the problems found out through such means as synthesizing the content in different areas, or making the connection of them with everyday phenomena and the learning of other school subjects. It is designed with the aim of encouraging initiatives for mathematical activities on the part of the students, as well as fostering students' ability to think, to judge, to express themselves and so forth. The implementation of "problem situation-based learning" should be properly positioned within the syllabus for each grade.