経済学と数学

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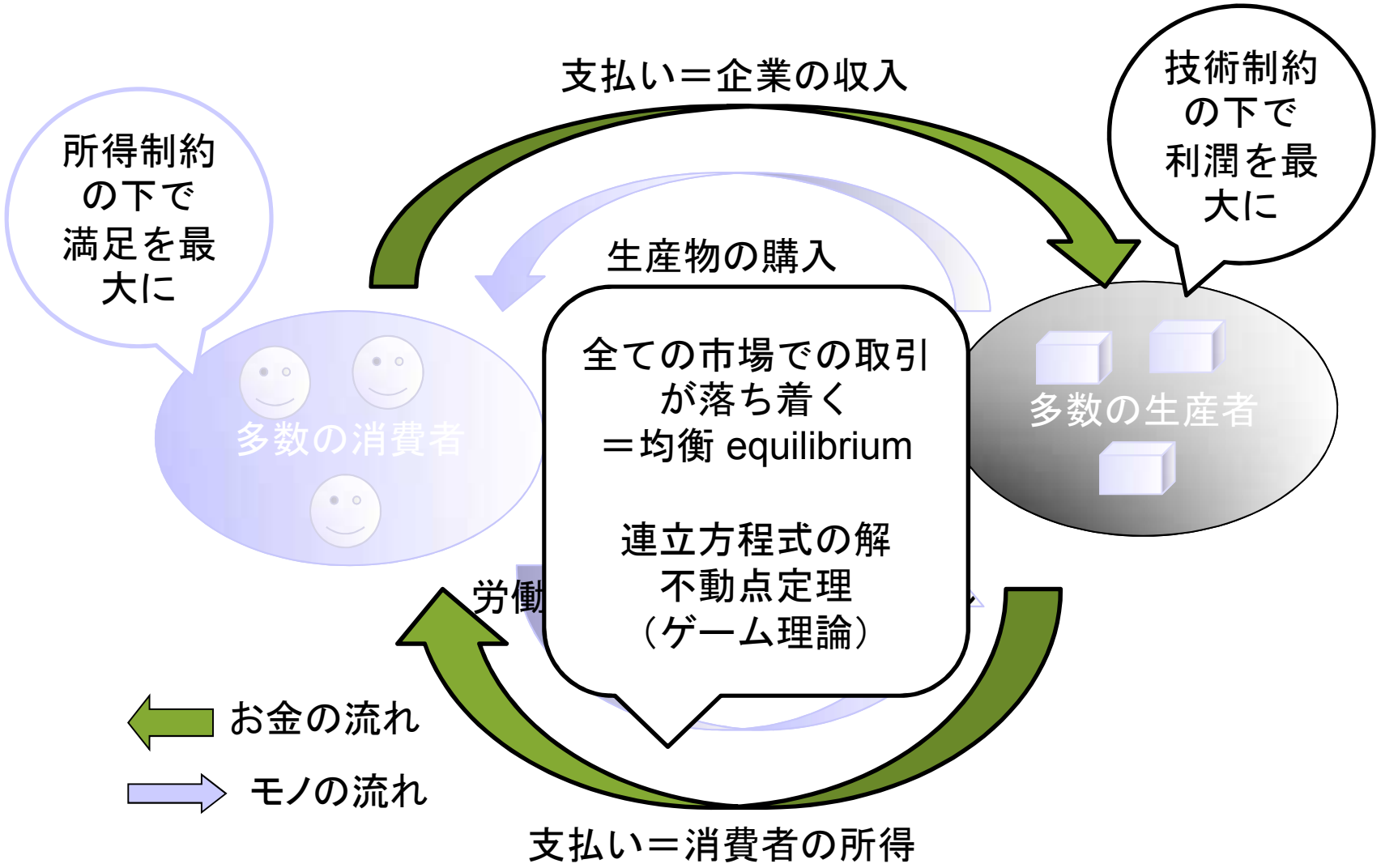
（近代）経済学の問題意識

- 資源がうまく配分されていない
- しかし、各意思決定者の自発的行動を尊重する

- 資源配分のメカニズムを知り（力学のようなもの）
- ルールを設計する(new!)

資源配分メカニズムの基本モデル



What Mathematics (Innovation) is needed in Economics

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Individual Consumer/Firm's Problem

- Given the prices of n goods and services $(p_1, p_2, \ldots, p_n)$
- Consumers and firms conduct constrained optimization to maximize her/his "satisfaction", profit etc. subject to budget or technological constraint.

$$\max_{(x_{1i}, x_{2i}, \ldots, x_{ni})} f_i(x_{1i}, x_{2i}, \ldots, x_{ni})$$

subject to
(consumer) $\sum_{k=1}^{n} p_k x_{ki} \leqq$ income
(firm) $g_i(x_{1i}, x_{2i}, \ldots, x_{ni}) \leqq 0$ (technological limitation)

- Solution: individual demand functions and supply functions

Prediction: Market Equilibrium

Find (hopefully unique) $(p_1, p_2, \ldots, p_n) \geqq 0$ such that all markets clear:

$$\sum_{i \in C} d_{1i}(p_1, p_2, \ldots, p_n) = (\leqq) \sum_{j \in F} s_{1j}(p_1, p_2, \ldots, p_n);$$

$$\sum_{i \in C} d_{2i}(p_1, p_2, \ldots, p_n) = (\leqq) \sum_{j \in F} s_{2j}(p_1, p_2, \ldots, p_n);$$

$$\ldots$$

$$\sum_{i \in C} d_{ni}(p_1, p_2, \ldots, p_n) = (\leqq) \sum_{j \in F} s_{nj}(p_1, p_2, \ldots, p_n).$$

Fixed Point Theorems.

Extension 1: Long Horizon

Decision-makers maximize long-term objectives.
Currently popular method: discounted dynamic programming

$$\max_{a_1,a_2,\ldots} \sum_{t=1}^{\infty} \delta^{t-1} f_i(a_t; s_t) \qquad (\text{or } \int_0^{\infty} e^{-rt} f_i(a_t; s_t) dt)$$

a_t: action by i at t
s_t: "state" at t, which can depend on the entire history

possibly with some constraints on actions
e.g., $a_t \leqq A - a_{t-1} + k_{t-1}$ (consumption not more than available amount)

More general methods desirable (not stationary discounting, more complex constraints allowed)

Extension 2: Stochasticity or Probability

- Stochastic shocks to consumers' tastes, technology, exchange rates, etc.
- Among firms, need to anticipate rival firms' actions, but you cannot be sure....

$$\max_{(x_{1i},x_{2i},\ldots,x_{ni})} E[f_i(x_{1i}, x_{2i}, \ldots, x_{ni})]$$

or

$$\max_{a_1,a_2,\ldots} E[\sum_{t=1}^{\infty} \delta^{t-1} f_i(a_t; s_t)]$$

Stochastic processes

Discrete Mathematics

- Optimization over integers
- Matching problems
  E.g., School choice, dormitory assignments, kidney exchange.
  Independent ranking among participants, ok. (Gale-Shapley, Top-Trading cycles.)
  - Inter-dependent rankings?
    Pupils care not just schools but also classmates.
  - Lying
    Pupils may not submit their true wish.

  Known algorithms may not work.